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FREE TRANSVERSE VIBRATIONS OF AN ELASTICALLY CONNECTED COMPLEX BEAM–STRING SYSTEM

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The present work is devoted to theoretical vibration analysis of a complex mixed continuous system. Undamped free transverse vibrations of an elastically connected beam–string system are considered. Solutions of the problem are formulated by using the modal expansion method. Two infinite sequences of the natural frequencies corresponding to two possible kinds of vibration motions: synchronous and asynchronous are determined. In a numerical example, illustrating the theory presented, the effect of string tension force on the natural frequencies of the system is investigated in detail.

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1. INTRODUCTION

Fundamental one- and two-dimensional simple continuous systems, such as a string, beam, membrane, plate and shell are usually used for modelling real mechanical structures. Connecting these simple systems by constraints of different type, interesting and technically important complex continuous systems are obtained. An elastically connected double-solid system is the simplest model of such a system, which is composed of two elastic solids attached continuously by a Winkler elastic layer [1]. The vibration analysis of complex continuous systems is of great theoretical and practical importance, and has a wide application in civil and mechanical engineering [1–8]. Reference [1] is entirely devoted to developing the general transverse vibration theory of some typical complex systems, namely: double-string, double-beam, double membrane and double-plate systems.

In the present paper, the mixed complex system consisting of a beam and a string continuously joined by means of a linear elastic element is considered. Analogous one-dimensional systems of two beams or two strings have been investigated by many authors, Seelig and Hoppmann [2], Saito and Chonan [3, 4], Kozlov [5], Kashin [6], Rao [7], Oniszczuk [1, 8–20], Irie *et al.* [21], Hamada *et al.* [22, 23], Yoshi and Upadhya [24], Vu [25], Frostig and Baruch [26], Kukla and Skalmierski [27], Sakiyama *et al.* [28], Lueschen and Bergman [29], Cabańska-Płaczkiewicz [30–32], Kukla [33], and Vu *et al.* [34], among others. Their considerations have referred to the various aspects of free vibrations for these interesting systems.

In this report, undamped free transverse vibrations of the title system [35] are analyzed and complete exact theoretical solutions of the problem are formulated.

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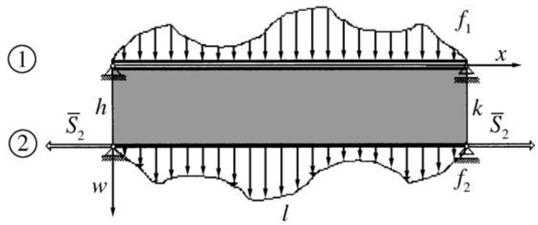

Figure 1. The general physical model of an elastically connected complex beam–string system.

2. FORMULATION OF THE PROBLEM

The scheme of the vibratory system under consideration is depicted in Figure 1. An elastically connected beam–string system consists of two parallel, one-dimensional continuous elastic bodies, which are a beam and a string attached to each other by a Winkler elastic layer. This classical foundation model is modelled as an infinite number of closely spaced independent (unconnected) linear massless springs. A uniform layer is characterized by one constant parameter, which is the stiffness modulus k (the Winkler foundation modulus). Both beam and string have the same length, and are simply supported at their ends. It is assumed that the beam is slender, prismatic and homogeneous. The string is uniform, homogeneous and is stretched under suitable constant tension. In the general case, it is also assumed that the system is subjected to transverse arbitrarily distributed continuous loads. Small undamped vibrations of the system are considered.

Applying the Bernoulli–Euler beam theory, transverse vibrations of an elastically connected beam–string system can be described by the following non-homogeneous partial differential equations [1, 35]:

$$m_1\ddot{w}_1 + K_1 w_1^{iv} + k(w_1 - w_2) = f_1, \qquad m_2\ddot{w}_2 - S_2 w_2'' + k(w_2 - w_1) = f_2, \tag{1}$$

where $w_i = w_i(x, t)$ is the transverse beam (string) deflection; x, t are the spatial co-ordinate and the time, respectively; $m_i = \rho_i F_i$, ρ_i is the mass density and F_i is the cross-sectional area of the beam (string); $K_1 = E_1 J_1$ is the flexural rigidity of the beam, E_1 is Young's modulus and J_1 is the moment of inertia of the beam cross-section; h, k are the thickness and stiffness modulus of a Winkler elastic layer, respectively; S_2 is the tension of the string; $f_i = f_i(x, t)$ is the distributed exciting load; l is the length of the beam (string); $\dot{w}_i = \partial w_i/\partial t$, $w_i' = \partial w_i/\partial x$, $i = 1, 2$. It is evident that subscripts 1 and 2 refer to the beam and string respectively.

The boundary conditions for a simply supported beam and string are as follows:

$$w_1(0, t) = w_1''(0, t) = w_1(l, t) = w_1''(l, t) = 0, \qquad w_2(0, t) = w_2(l, t) = 0. \tag{2}$$

The initial conditions for this problem are assumed in the general form

$$w_i(x, 0) = w_{i0}(x), \qquad \dot{w}_i(x, 0) = v_{i0}(x), \quad i = 1, 2. \tag{3}$$

3. SOLUTION OF THE FREE VIBRATION PROBLEM

The governing homogeneous partial differential equations for free vibrations of a beam–string system (see Figure 2) are as follows:

$$m_1\ddot{w}_1 + K_1 w_1^{iv} + k(w_1 - w_2) = 0, \qquad m_2\ddot{w}_2 - S_2 w_2'' + k(w_2 - w_1) = 0. \tag{4}$$

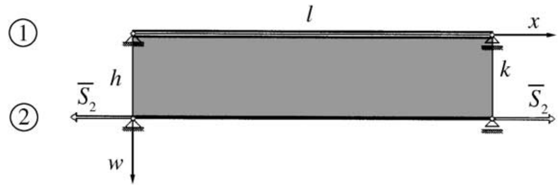

Figure 2. The analyzed model of an elastically connected complex beam–string system.

Free vibrations of the system considered are determined by using the classical modal expansion method. Thus, general solutions of equations (4) satisfying the boundary conditions (2) are assumed to be in the form

$$w_1(x,t) = \sum_{n=1}^{\infty} X_n(x) S_{1n}(t) = \sum_{n=1}^{\infty} \sin(k_n x) S_{1n}(t),$$
$$w_2(x,t) = \sum_{n=1}^{\infty} X_n(x) S_{2n}(t) = \sum_{n=1}^{\infty} \sin(k_n x) S_{2n}(t), \tag{5}$$

where $S_{in}(t)$ $(i = 1, 2)$ are the unknown time functions and

$$X_n(x) = \sin(k_n x), \quad k_n = l^{-1} n\pi, \quad n = 1, 2, 3, \dots. \tag{6}$$

$X_n(x)$ is the known mode shape function for a simply supported single beam and string. Substituting expressions (5) into equations (4) results in the relations:

$$\sum_{n=1}^{\infty} [m_1 \ddot{S}_{1n} + (K_1 k_n^4 + k) S_{1n} - k S_{2n}] X_n = 0,$$

$$\sum_{n=1}^{\infty} [m_2 \ddot{S}_{2n} + (S_2 k_n^2 + k) S_{2n} - k S_{1n}] X_n = 0$$

from which a set of ordinary differential equations for unknown time functions is obtained

$$\ddot{S}_{1n} + \omega_{11n}^2 S_{1n} - \omega_{10}^2 S_{2n} = 0, \quad \ddot{S}_{2n} + \omega_{22n}^2 S_{2n} - \omega_{20}^2 S_{1n} = 0, \tag{7}$$

where

$$\omega_{11n}^2 = (K_1 k_n^4 + k) m_1^{-1}, \qquad \omega_{22n}^2 = (S_2 k_n^2 + k) m_2^{-1},$$

$$\omega_{i0}^2 = k m_i^{-1}, \quad \omega_{120}^4 = \omega_{10}^2 \omega_{20}^2 = k^2 (m_1 m_2)^{-1}, \quad i = 1, 2.$$

The solutions of equations (7) have the form

$$S_{1n}(t) = C_n \mathrm{e}^{\mathrm{i}\omega_n t}, \qquad S_{2n}(t) = D_n \mathrm{e}^{\mathrm{i}\omega_n t}, \quad \mathrm{i} = (-1)^{1/2}, \tag{8}$$

where ω_n denotes the natural frequency of the system. After introducing them into equations (7) one obtains a homogeneous set of algebraic equations for unknown constants

C_n, D_n:

$$(\omega_{11n}^2 - \omega_n^2)C_n - \omega_{10}^2 D_n = 0, \quad (\omega_{22n}^2 - \omega_n^2)D_n - \omega_{20}^2 C_n = 0. \tag{9}$$

For non-trivial solutions of equations (9), the cardinal determinant of the system coefficient matrix must vanish. This gives the following frequency equation:

$$\omega_n^4 - (\omega_{11n}^2 + \omega_{22n}^2)\omega_n^2 + (\omega_{11n}^2\omega_{22n}^2 - \omega_{120}^4) = 0, \tag{10}$$

$$\omega_n^4 - [(K_1k_n^4 + k)m_1^{-1} + (S_2k_n^2 + k)m_2^{-1}]\omega_n^2$$
$$+ k_n^2[K_1S_2k_n^4 + k(K_1k_n^2 + S_2)](m_1m_2)^{-1} = 0. \tag{11}$$

The frequency equations (10, 11) have two real, positive roots $\omega_{1,2n}^2$ [35]:

$$\omega_{1,2n}^2 = 0{\cdot}5\{(\omega_{11n}^2 + \omega_{22n}^2) \mp [(\omega_{11n}^2 - \omega_{22n}^2)^2 + 4\omega_{120}^4]^{1/2}\}, \quad \omega_{1n} < \omega_{2n}. \tag{12}$$

Then, two infinite sequences of natural frequencies ω_{1n} and ω_{2n} are obtained:

$$\omega_{1,2n}^2 = 0{\cdot}5\{[(K_1k_n^4 + k)m_1^{-1} + (S_2k_n^2 + k)m_2^{-1}] \mp ([(K_1k_n^4 + k)m_1^{-1}$$
$$+ (S_2k_n^2 + k)m_2^{-1}]^2 - 4k_n^2(m_1m_2)^{-1}[K_1S_2k_n^4 + k(K_1k_n^2 + S_2)])^{1/2}\}. \tag{13}$$

Now solutions (8) may be written as

$$S_{1n}(t) = C_{1n}e^{i\omega_{1n}t} + C_{2n}e^{-i\omega_{1n}t} + C_{3n}e^{i\omega_{2n}t} + C_{4n}e^{-i\omega_{2n}t},$$

$$S_{2n}(t) = D_{1n}e^{i\omega_{1n}t} + D_{2n}e^{-i\omega_{1n}t} + D_{3n}e^{i\omega_{2n}t} + D_{4n}e^{-i\omega_{2n}t}.$$

Introducing trigonometric functions, the above unknown time functions are transformed into the following form:

$$S_{1n}(t) = \sum_{i=1}^{2} T_{in}(t) = \sum_{i=1}^{2} [A_{in}\sin(\omega_{in}t) + B_{in}\cos(\omega_{in}t)],$$
$$S_{2n}(t) = \sum_{i=1}^{2} a_{in}T_{in}(t) = \sum_{i=1}^{2} [A_{in}\sin(\omega_{in}t) + B_{in}\cos(\omega_{in}t)]a_{in}, \tag{14}$$

where

$$T_{in}(t) = A_{in}\sin(\omega_{in}t) + B_{in}\cos(\omega_{in}t), \quad i = 1, 2, \tag{15}$$

$$a_{in} = (K_1k_n^4 + k - m_1\omega_{in}^2)k^{-1} = k(S_2k_n^2 + k - m_2\omega_{in}^2)^{-1}$$
$$= \omega_{10}^{-2}(\omega_{11n}^2 - \omega_{in}^2) = \omega_{20}^2(\omega_{22n}^2 - \omega_{in}^2)^{-1}. \tag{16}$$

The mode shape coefficients a_{in} (16) can be presented as

$$a_{1,2n} = 0{\cdot}5\omega_{10}^{-2}\{(\omega_{11n}^2 - \omega_{22n}^2) \pm [(\omega_{11n}^2 - \omega_{22n}^2)^2 + 4\omega_{120}^4]^{1/2}\}, \quad a_{1n} > 0, \quad a_{2n} < 0,$$

$$a_{1n}a_{2n} = -m_1 m_2^{-1} = -M_1 M_2^{-1} = -\omega_{10}^{-2}\omega_{20}^2, \quad M_i = m_i l = \rho_i F_i l,$$

which allows one to easily prove that the coefficient a_{1n}, dependent on the lower natural frequency ω_{1n}, is always positive while a_{2n}, dependent on the higher frequency ω_{2n}, is always negative.

Finally, the free transverse vibrations of a complex elastic beam–string system are expressed by the following formulae:

$$w_1(x,t) = \sum_{n=1}^{\infty} X_n(x) S_{1n}(t) = \sum_{n=1}^{\infty} X_n(x) \sum_{i=1}^{2} T_{in}(t) = \sum_{n=1}^{\infty}\sum_{i=1}^{2} X_{1in}(x) T_{in}(t)$$

$$= \sum_{n=1}^{\infty} \sin(k_n x) \sum_{i=1}^{2} [A_{in}\sin(\omega_{in} t) + B_{in}\cos(\omega_{in} t)], \tag{17}$$

$$w_2(x,t) = \sum_{n=1}^{\infty} X_n(x) S_{2n}(t) = \sum_{n=1}^{\infty} X_n(x) \sum_{i=1}^{2} a_{in} T_{in}(t) = \sum_{n=1}^{\infty}\sum_{i=1}^{2} a_{in} X_n(x) T_{in}(t)$$

$$= \sum_{n=1}^{\infty}\sum_{i=1}^{2} X_{2in}(x) T_{in}(t) = \sum_{n=1}^{\infty} \sin(k_n x) \sum_{i=1}^{2} [A_{in}\sin(\omega_{in} t) + B_{in}\cos(\omega_{in} t)] a_{in},$$

where

$$X_{1in}(x) = X_n(x) = \sin(k_n x), \quad X_{2in}(x) = a_{in} X_n(x) = a_{in}\sin(k_n x). \tag{18}$$

The functions $X_{1in}(x)$ and $X_{2in}(x)$ are the natural mode shapes of vibration of a system corresponding to two sequences of the natural frequencies ω_{in}. An elastically connected complex beam–string system executes two kinds of vibrating motions: synchronous vibrations ($a_{1n} > 0$) with lower natural frequencies ω_{1n} and asynchronous vibrations ($a_{2n} < 0$) with higher frequencies ω_{2n}. The general mode shapes of vibration (18) are exactly the same as the natural mode shapes determined for a double string [18], and a simply supported double-beam system [19]. It can also be shown that the nature of free vibrations is analogous and the mathematical form of the corresponding solutions is identical for all these three systems as a consequence of governing the same boundary conditions.

Now the initial-value problem is considered, to determine the final form of the free vibrations. The unknown constants A_{in} and B_{in} are evaluated from the assumed initial conditions (3) using the orthogonality property of the mode shape functions. In this case the classical orthogonality condition is governed

$$\int_0^l X_m X_n \,\mathrm{d}x = \int_0^l \sin(k_m x)\sin(k_n x)\,\mathrm{d}x = c\delta_{mn}, \tag{19}$$

$$c = c_n^2 = \int_0^l X_n^2 \,\mathrm{d}x = \int_0^l \sin^2(k_n x)\,\mathrm{d}x = 0{\cdot}5l,$$

where δ_{mn} is the Kronecker delta function: $\delta_{mn} = 0$ *for* $m \neq n$ and $\delta_{mn} = 1$ *for* $m = n$. Substituting solutions (17) into the initial conditions (3) yields

$$w_{10} = \sum_{n=1}^{\infty} X_n \sum_{i=1}^{2} B_{in}, \qquad v_{10} = \sum_{n=1}^{\infty} X_n \sum_{i=1}^{2} \omega_{in} A_{in},$$

$$w_{20} = \sum_{n=1}^{\infty} X_n \sum_{i=1}^{2} a_{in} B_{in}, \qquad v_{20} = \sum_{n=1}^{\infty} X_n \sum_{i=1}^{2} a_{in} \omega_{in} A_{in}.$$

Multiplying the above relations by the eigenfunction X_m, then integrating them with respect to x form 0 to l, and applying the orthogonality condition (19) produces a system of algebraic equations allowing one to determine the following formulas for the unknown constants:

$$A_{1n} = (c_{1n}\omega_{1n})^{-1} \int_0^l (a_{2n} v_{10} - v_{20}) \sin(k_n x)\, \mathrm{d}x, \qquad B_{1n} = c_{1n}^{-1} \int_0^l (a_{2n} w_{10} - w_{20}) \sin(k_n x)\, \mathrm{d}x,$$

$$A_{2n} = (c_{2n}\omega_{2n})^{-1} \int_0^l (a_{1n} v_{10} - v_{20}) \sin(k_n x)\, \mathrm{d}x, \qquad B_{2n} = c_{2n}^{-1} \int_0^l (a_{1n} w_{10} - w_{20}) \sin(k_n x)\, \mathrm{d}x, \tag{20}$$

where

$$c_{1n} = -c_{2n} = (a_{2n} - a_{1n})c = 0{\cdot}5 l \omega_{10}^{-2} (\omega_{1n}^2 - \omega_{2n}^2).$$

It is seen that the free vibration analysis performed in the present paper for a beam–string system is analogous to that for a double string [18], and simply supported double-beam system [19].

4. NUMERICAL EXAMPLE

The theoretical analysis presented is illustrated by a numerical example, in which the effect of a string tension force S_2 on the natural frequencies of the system is mainly investigated. The influence of a beam flexural rigidity K_1 is also taken into account.

The following values of the parameters characterizing the physical and geometrical properties of the beam–string system are used in the numerical calculations:

$$E_1 = 1 \times 10^{10}\ \mathrm{N/m^2}, \quad J_1 = 1 \times 10^{-7};\ 1 \times 10^{-6};\ 1 \times 10^{-5};\ 1 \times 10^{-4};\ 1 \times 10^{-3}\ \mathrm{m^4},$$

$$K_1 = E_1 J_1 = 1 \times 10^3;\ 1 \times 10^4;\ 1 \times 10^5;\ 1 \times 10^6;\ 1 \times 10^7\ \mathrm{N\,m^2}, \quad l = 10\,\mathrm{m},$$

$$\rho_1 = 2 \times 10^3\ \mathrm{kg/m^3}; \quad F_1 = 5 \times 10^{-2}\ \mathrm{m^2}, \quad m_1 = \rho_1 F_1 = 1 \times 10^2\ \mathrm{kg/m},$$

$$\rho_2 = 1 \times 10^3\ \mathrm{kg/m^3}; \quad F_2 = 1 \times 10^{-2}\ \mathrm{m^2}, \quad m_2 = \rho_2 F_2 = 10\ \mathrm{kg/m},$$

$$k = 1 \times 10^3\ \mathrm{N/m^2}, \quad S_2 = 0;\ 1000;\ 2000;\ 3000;\ 4000;\ 5000\ \mathrm{N}.$$

Free vibrations of the system discussed are described by relations (17):

$$w_1(x,t) = \sum_{n=1}^{\infty} \sum_{i=1}^{2} X_{1in}(x)\, T_{in}(t)$$

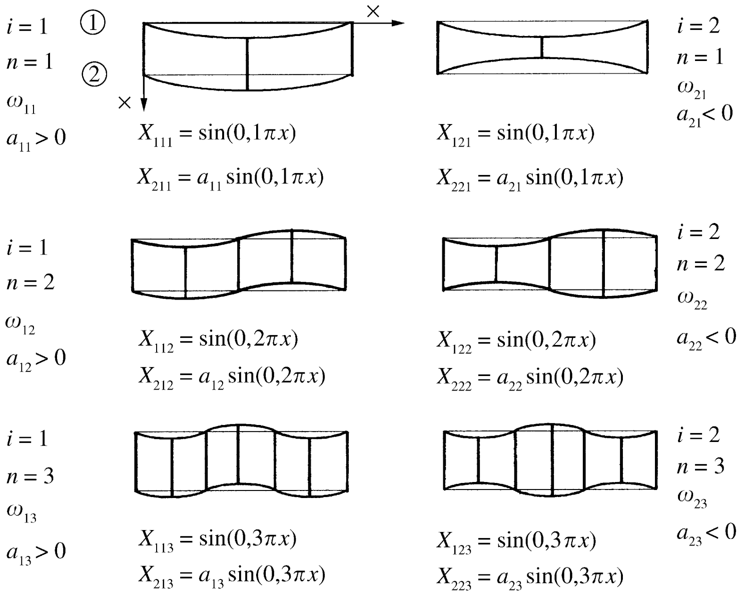

Figure 3. The natural model shapes of vibration of an elastically connected complex beam–string system corresponding to the first three pairs of natural frequencies of the system. The mode shapes for $i = 1$ and 2 express the synchronous ($a_{1n} > 0$, ω_{1n}) and asynchronous ($a_{2n} < 0$, ω_{2n}) free vibrations respectively.

Table 1

Natural frequencies of beam–string system ω_{in} (s^{-1}); $K_1 = 1 \times 10^3$ (N m^2)

S_2	ω_{11}	ω_{21}	ω_{12}	ω_{22}	ω_{13}	ω_{23}
0	0·297	10·489	1·190	10·495	2·669	10·525
1000	0·595	10·911	2·042	12·365	3·497	13·946
2000	1·277	11·323	2·405	13·592	3·754	16·776
3000	1·499	11·724	2·614	14·939	3·879	19·218
4000	1·668	12·116	2·750	16·184	3·952	21·391
5000	1·803	12·498	2·846	17·345	4·000	23·367

Table 2

Natural frequencies of beam–string system ω_{in} (s^{-1}); $K_1 = 1 \times 10^4$ (N m^2)

S_2	ω_{11}	ω_{21}	ω_{12}	ω_{22}	ω_{13}	ω_{23}
0	0·941	10·492	3·737	10·565	7·896	11·250
1000	1·314	10·914	4·171	12·152	8·929	14·071
2000	1·565	11·325	4·396	13·610	9·151	16·817
3000	1·753	11·727	4·531	14·950	9·238	19·236
4000	1·901	12·118	4·621	16·191	9·284	21·401
5000	2·022	12·499	4·686	17·350	9·312	23·373

TABLE 3

Natural frequencies of beam–string system ω_{in} (s^{-1}); $K_1 = 1 \times 10^5$ (N m^2)

S_2	ω_{11}	ω_{21}	ω_{12}	ω_{22}	ω_{13}	ω_{23}
0	2·963	10·534	9·342	13·363	9·928	28·292
1000	3·123	10·948	10·881	13·672	13·682	28·296
2000	3·253	11·353	11·837	14·307	16·605	28·301
3000	3·356	11·750	12·289	15·273	19·083	28·307
4000	3·450	12·137	12·492	16·362	21·270	28·318
5000	3·526	12·516	12·596	17·453	23·244	28·355

TABLE 4

Natural frequencies of beam–string system ω_{in} (s^{-1}); $K_1 = 1 \times 10^6$ (N m^2)

S_2	ω_{11}	ω_{21}	ω_{12}	ω_{22}	ω_{13}	ω_{23}
0	8·447	11·642	9·966	39·487	9·994	88·883
1000	8·774	11·844	11·780	39·487	13·737	88·883
2000	9·020	12·074	13·350	39·488	16·659	88·883
3000	9·219	12·330	14·754	39·488	19·140	88·883
4000	9·380	12·606	16·036	39·488	21·335	88·883
5000	9·510	12·897	17·222	39·488	23·324	88·883

TABLE 5

Natural frequencies of beam–string system ω_{in} (s^{-1}); $K_1 = 1 \times 10^7$ (N m^2)

S_2	ω_{11}	ω_{21}	ω_{12}	ω_{22}	ω_{13}	ω_{23}
0	9·943	31·388	9·997	124·842	9·999	280·894
1000	10·427	31·388	11·807	124·842	13·741	280·894
2000	10·890	31·388	13·375	124·842	16·663	280·894
3000	11·330	31·388	14·777	124·842	19·143	280·894
4000	11·760	31·389	16·058	124·842	21·338	280·894
5000	12·172	31·389	17·243	124·842	23·326	280·894

$$= \sum_{n=1}^{\infty} \sin(k_n x) \sum_{i=1}^{2} [A_{in} \sin(\omega_{in} t) + B_{in} \cos(\omega_{in} t)],$$

$$w_2(x, t) = \sum_{n=1}^{\infty} \sum_{i=1}^{2} X_{2in}(x) T_{in}(t)$$

$$= \sum_{n=1}^{\infty} \sin(k_n x) \sum_{i=1}^{2} [A_{in} \sin(\omega_{in} t) + B_{in} \cos(\omega_{in} t)] a_{in}.$$

The general natural mode shapes of vibration $X_{1in}(x)$ and $X_{2in}(x)$ are (18)

$$X_{1in}(x) = X_n(x) = \sin(k_n x), \quad X_{2in}(x) = a_{in} X_n(x) = a_{in} \sin(k_n x),$$

TABLE 6

Coefficients of natural mode shapes a_{in}; $K_1 = 1 \times 10^3$ (N m^2)

S_2	a_{11}	a_{21}	a_{12}	a_{22}	a_{13}	a_{23}
0	1·00	−9·99	1·01	−9·87	1·08	−9·90
1000	0·92	−10·89	0·74	−14·15	0·56	−17·67
2000	0·85	−11·80	0·58	−17·31	0·38	−26·37
3000	0·79	−12·73	0·47	−21·16	0·28	−35·15
4000	0·73	−13·68	0·40	−25·02	0·23	−43·96
5000	0·69	−14·62	0·34	−28·95	0·19	−52·83

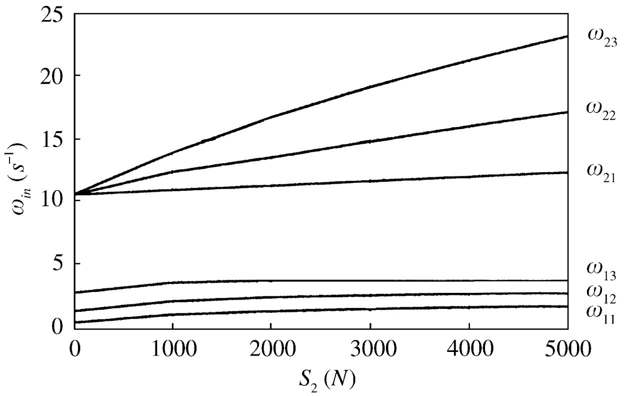

Figure 4. The natural frequencies of beam–string system ω_{in} ($i = 1, 2$; $n = 1, 2, 3$) as a function of string tension force S_2 for a beam flexural rigidity $K_1 = 1 \times 10^3$ N m^2.

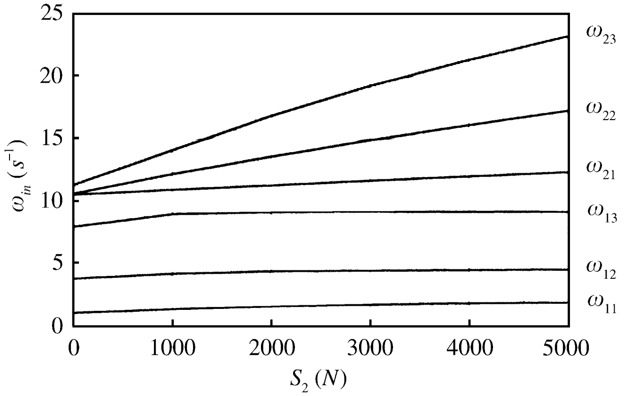

Figure 5. The natural frequencies of beam–string system ω_{in} ($i = 1, 2$; $n = 1, 2, 3$) as a function of string tension force S_2 for a beam flexural rigidity $K_1 = 1 \times 10^4$ N m^2.

where

$$a_{1n} > 0, \qquad a_{2n} < 0, \qquad k_n = l^{-1} n\pi, \quad i = 1, 2.$$

The mode shapes corresponding to the first three pairs of natural frequencies are shown in Figure 3. The mode shapes for $i = 1$ and 2 express the synchronous ($a_{1n} > 0$, ω_{1n}) and

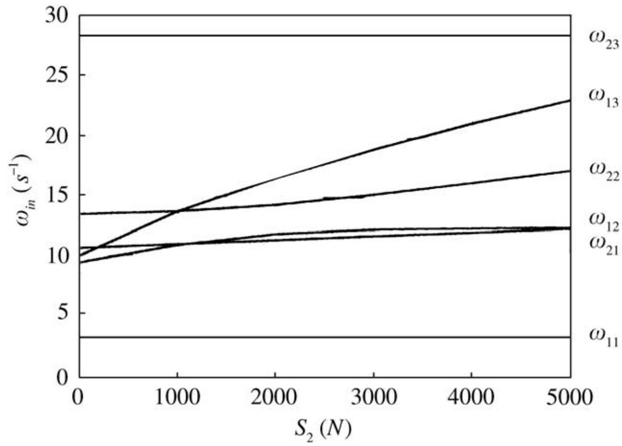

Figure 6. The natural frequencies of beam–string system ω_{in} ($i = 1, 2$; $n = 1, 2, 3$) as a function of string tension force S_2 for a beam flexural rigidity $K_1 = 1 \times 10^5$ N m^2.

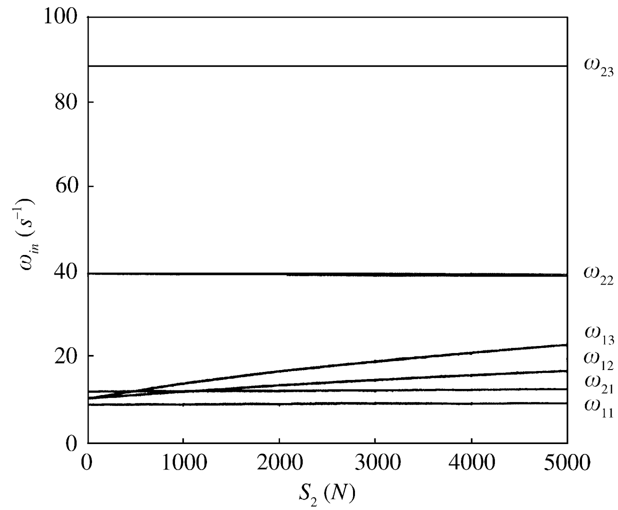

Figure 7. The natural frequencies of beam–string system ω_{in} ($i = 1, 2$; $n = 1, 2, 3$) as a function of string tension force S_2 for a beam flexural rigidity $K_1 = 1 \times 10^6$ N m^2.

asynchronous ($a_{2n} < 0$, ω_{2n}) free vibrations of the system respectively. The natural frequencies ω_{in} and mode shape coefficients a_{in} are evaluated from expressions (12, 13) and (16) as functions of a string tension force S_2 for different values of a beam flexural rigidity K_1. Results of the calculations of ω_{in} and a_{in} (as an instance, only for $K_1 = 1 \times 10^3$ N m^2) for $i = 1, 2$ and $n = 1, 2, 3$ are presented in Tables 1–6 and in Figures 4–9.

Analyzing the effect of the string tension force on the natural frequencies of an elastically connected beam–string system, the following conclusions can be drawn. In general, there is an evident tendency to increase the natural frequencies ω_{in} (13) in the case of increasing the tension force S_2, but the influence of S_2 on the particular frequencies is different and strongly depends on the beam flexural rigidity K_1. For small values of K_1 ($= 1 \times 10^3$; 1×10^4 N m^2) the effect of S_2 on the asynchronous frequencies ω_{2n} is greater than on the synchronous ones ω_{1n} (see Tables 1 and 2, and Figures 4 and 5). The situation decidedly changes in the case of large values of K_1 ($= 1 \times 10^6$; 1×10^7 N m^2), then the frequencies ω_{1n}

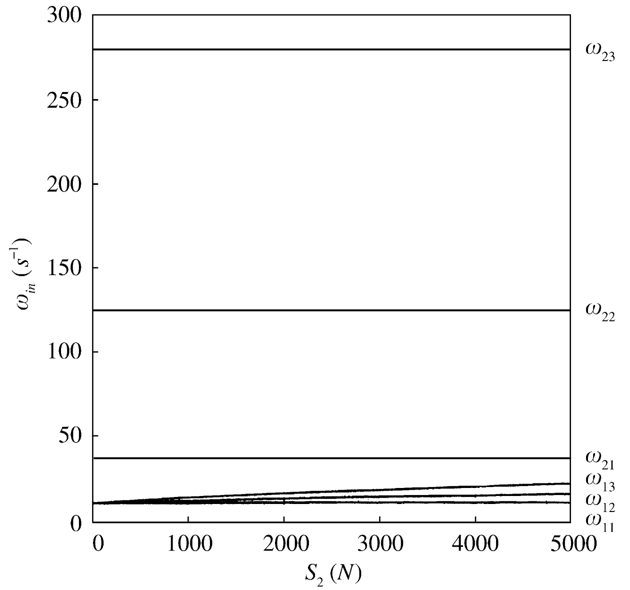

Figure 8. The natural frequencies of beam–string system ω_{in} ($i = 1, 2$; $n = 1, 2, 3$) as a function of string tension force S_2 for a beam flexural rigidity $K_1 = 1 \times 10^7$ N m^2.

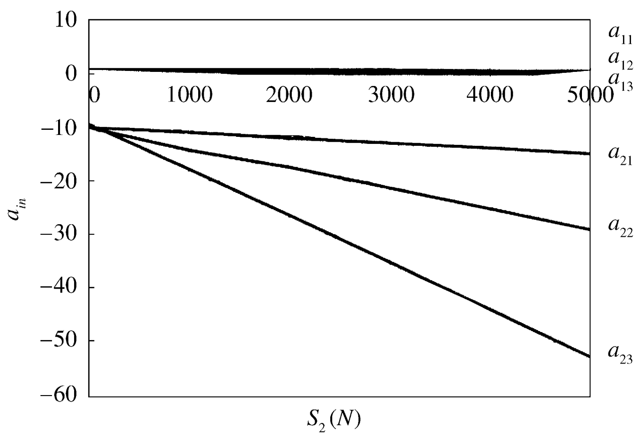

Figure 9. The natural mode shape coefficients of beam–string system a_{in} ($i = 1, 2$; $n = 1, 2, 3$) as a function of string tension force S_2 for a beam flexural rigidity $K_1 = 1 \times 10^3$ N m^2.

significantly depend on S_2, while all frequencies ω_{2n} remain almost constants (they are independent of S_2, in principle) (see Tables 4 and 5, and Figures 7 and 8).

It is important to note, that for this interesting vibratory system, the possibility of changing the natural frequencies by a variation of only the string tension forces exists. It is of great worth that the structural parameters of the system need not be changed. This fact can have significance in practical applications of such mixed complex systems. Choosing properly tension forces of a string one obtains desirable values of the system frequencies in certain limited domains, which allows one to avoid resonance phenomena or to generate a dynamic vibration absorption phenomenon, making it possible to suppress excessive forced vibration amplitudes [1, 20, 23, 34].

5. CONCLUSIONS

This paper deals with the undamped free transverse vibration theory of an elastically connected complex beam–string continuous system. General solutions of the homogeneous partial differential equations of motion are formulated by using the classical modal expansion method. Two infinite sequences of the natural frequencies ω_{1n} and ω_{2n} ($\omega_{1n} < \omega_{2n}$) corresponding to two sequences of mode shape functions expressing the synchronous ($a_{1n} > 0$, ω_{1n}) and asynchronous ($a_{2n} < 0$, ω_{2n}) free vibrations of the system are determined. The initial-value problem is also considered to find the final form of free vibrations. An interesting feature of the beam–string system considered should be emphasized, namely, its natural frequencies can be varied by a change of string tension force only, without the necessity of variation of the other parameters characterizing the physical and geometrical properties of the system. This possibility can be of great practical importance. It is easy to show, that the nature of free vibrations for a beam–string, double string [18], and of a simply supported double-beam system [19] is similar.

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